Applying Bruner's Theory of Representation to Teach Pre-Algebra and Algebra Concepts to Community College Students Using Virtual Manipulatives

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Abstract

This study investigated the impact of the use of virtual manipulatives on community college remedial students' attitudes, confidence and achievement in the learning of pre-algebra and algebra concepts. Since urban community college remedial students lack fundamental basic arithmetic and algebra skills similar to middle school students, we combined the use of computer applications with Bruner's theory of three stages of representation to an experimental group while the control group was taught without computers.

A primary finding seems to be that the virtual manipulatives appear to be more useful in teaching pre-algebra remedial courses than in algebra remedial courses. At both levels, experimental group students overcame their initial mathematics misconceptions with less difficulty than the students in the control group. They found the classes with virtual manipulatives very exciting as the computer software provided them with many new practice exercises and instant feedback. At the college level, virtual manipulatives could play a significant role in the teaching of remedial math classes (pre-algebra and algebra) to community college students. The use of technology-rich and easy-to-use materials can also be appealing to college instructors of these classes, more so than hands-on manipulatives.

1. Background

The purpose of this study was to investigate the impact of the use of virtual manipulatives on community college remedial students' attitudes, confidence and achievement in the learning of pre-algebra and algebra concepts. Stipulating that community college remedial students, who lack fundamental basic arithmetic and algebra skills similar to middle school students, can learn according to Bruner's theory of representation [6], three CUNY mathematics and mathematics education researchers designed modules related to the study of pre-algebra and algebra concepts with (experimental group) and without (control group) the use of virtual manipulatives.

The development of a plethora of hands-on and virtual manipulatives to teach mathematics concepts has flourished in the market and on the Internet ([4], [29], [5], [26]). Unfortunately, however, very limited research in the implementation of the teaching techniques that accompany the materials and of the cognitive issues related to their use has been conducted, especially at the college remedial level which has seen a large increase in the number of freshmen that enroll in remediation courses.

The continuous crisis in U. S. science and mathematics education at the elementary and secondary levels ([38], [27]) has indeed profoundly affected the postsecondary education level. The proportion of students performing at or under grade proficiency level in mathematics and science is alarming and increases from 4^{th} to 8^{th} grade, and again from 8^{th} to12^{th} grade [2]. The consequences are deeply felt when these students enter college. In fall 2000 for instance, some 76 percent of postsecondary institutions offered at least one remedial reading, writing, or mathematics course. Postsecondary transcripts of 1992 12^{th}-graders who enrolled in postsecondary education between 1992 and 2000 show that 61 percent of students who first attended a public 2-year and 25 percent who first attended a 4-year institution completed at least one remedial course at the postsecondary level [2].

Locally, at the City University of New York (CUNY), the situation is even more pronounced. Community college students represent 33% of the total CUNY student population, with most coming from New York City public schools. We conducted this research in parallel at two CUNY community colleges, thereafter referred to as CC1 and CC2. CC1 has a student enrollment of approximately 6000 students. Nearly half of them come from New York City public high schools while approximately 24 percent enter the college with a GED diploma. Nearly 80 percent of the incoming freshmen at CC1 are required to take at least one remedial course as determined by the college placement exam (COMPASS). CC2 enrolls approximately 1,100 freshmen each year. A large percentage of these students have to take remedial mathematics courses in either pre-algebra, algebra or in both. Students who have not mastered pre-algebra concepts – as documented by a grade lower than C – stand less than 12% chance of ever graduating from college (CC2 Director of Institutional Research and Testing).

Thus, mastery in pre-algebra seems to be an indicator of college success. To prepare students for credit-bearing courses in mathematics and in science, colleges nationwide employ different strategies, increasing hours of remediation being one of them. CC1 students requiring remediation in pre-algebra attend a semester-long course consisting of 4.5 hours of lecture and 1.5 hours of tutoring in the learning center each week. The comparable course at CC2 consists of 4 hours weekly lecture; tutoring at CC2 is available, but not mandatory. The challenge, however, is to find effective methods that address students' different learning styles and pace of learning. Frontal lecture, the traditional style most prevalent in American college mathematics classrooms is the least effective teaching method as far as concept understanding and retention [35], since it does not naturally take into account different students' learning styles and variation in students' cognitive abilities.

2. *Theoretical Framework*

The use of manipulatives in instruction is one strategy commonly used to encourage learners to become more actively involved in mathematics. Because manipulatives allow students to build up mental representations and acquire skills in using and modifying these representations and synthesizing new ones ([7]; [8]), they have been described and found by many to be the best approach to resolve the difficulties inherent in learning arithmetic and algebra concepts and processes ([6]; [10]; [14]; [15]; [22]; [20]; [21]; [23]). Whereas hands-on manipulatives are tactile and visual, virtual manipulatives are only visual. Virtual manipulatives are however also interactive: that is, the learner can manipulate the same objects and create the same mental representations of the objects using the computer mouse. In today's technology-enriched environment, it is even more appealing for college professors and college students to use computers rather than hands-on manipulatives.

The view that learning takes place when students create (construct) new mathematical knowledge by reflecting on their physical and mental actions is mostly derived from Piaget's descriptive theory of developmental stages and Bruner's [6] prescriptive theory of modes of representational thought ([41]). While Piaget suggests that intellectual development progresses through different stages in which construction precedes analysis, Bruner thought that learning by discovery involved an internal reorganization of previously known ideas and stipulated that children move through three modes or levels of representation as they learn. In the first or enactive level, the child needs action on materials to understand a concept. In the second or iconic level, the child creates mental representations of the objects but does not manipulate them directly; rather, the concept is represented pictorially. Finally, in the third or symbolic level, the child is strictly manipulating symbols and does not need to manipulate objects.

This view emphasizes the need for students to mentally represent mathematical concepts in general and algebraic concepts in particular. Mental or internal representations represent the essence of constructivism. Constructivists view the learning of mathematics as a process of building up mental representations and acquiring skills in using and modifying these representations, and synthesizing new ones [37]. Many ([6]; [10]) view the constructivist approach as the best way to resolve difficulties in the teaching and learning of mathematics, especially pre and algebra concepts. While testing the use of manipulatives to develop pre-algebra and algebra concepts has been carried often times for many decades now ([7], [15], [23]), the same could not be said about technology in mathematics, especially interactive technology, which is almost new [4], [5], [29].

Additionally, if investigating the *impact* of various teaching methods on student achievement is important, equally important is the study of such impact on affective variables such as students' attitudes toward and confidence in learning mathematics ([3]; [12]; [22]; [28; [32]; [33]; [36]; [39]). In middle schools, for instance, many students adopt the attitude that manipulatives are for young children and consequently block their full engagement in a situation that might otherwise be productive for learning ([30]).

Such "attitudinal interference" could however be minimized in college, when they are studying in different contexts with different personal expectations. For Friedman [13], how we educate our children may prove to be more important than how much we educate them. Students must be taught the love of learning: "Curiosity Quotient" plus "Passion Quotient" is greater than "Intelligence Quotient," (CQ + PQ > IQ), because curious, passionate kids are self-educators and self-motivators. Fennema and Sherman [12] contend that affective and attitudinal factors have an *impact* on the amount of effort one is willing to expend to learn mathematics. Affective factors may also influence decisions about taking additional mathematics courses in the future.

3. Research Design and Method

This research stipulates that community college remedial students can learn according to Bruner's theory of representation. Bruner's modes of representation were used to develop materials and teaching strategies. Numerical and variables concepts were made concrete through the use of virtual manipulatives and applied to an instructional treatment for community college remedial mathematics students. In this model, in the first (enactive) stage, students **virtually** manipulated algebra tiles, Cuisenaire rods, and pattern blocks using computers. In the second (iconic) stage, students made representations (drawings) of the ideas developed with the virtual tiles, rods, or blocks on handouts that were provided, thereby reducing the time they spent copying notes from the board. Finally, in the third (symbolic) stage, students analyzed their drawings and through the mental representations formed in the first two stages, developed and stated conjectures and important basic mathematical formulae, theorems, and processes for themselves and then practiced using them.

We hypothesized that, due to the intervention with virtual manipulatives:

1. Students in the experimental group will perform significantly better on achievement tests than students in the control group in all pre-algebra concepts.
2. Students in the experimental group will perform significantly better on achievement tests than students in the control group in all algebra concepts.
3. There will be significant improvement in attitudes toward mathematics from pre-post surveys with experimental group students.
4. Experimental group students will show significantly more improvement on tests of confidence in mathematical ability than control group students.

3.1 Participants

3.11 Instructional Faculty. This quasi-experimental research project took place during the spring 2010, fall 2010, and spring 2011 semesters at two CUNY community colleges, CC1 and CC2 (Table 3). Two of the three faculty each taught one experimental pre-algebra class (n=23 at CC1, n= 19 at CC2) and one experimental algebra (n=28 at CC1, n=23 at CC2) group and their corresponding control groups (Pre-algebra: n=22 at CC1, n= 24 at CC2; Algebra: n=28 at CC1, n= 25 at CC2) in their respective campuses as randomly assigned by their department chairs. The first instructor was a full time associate professor of mathematics at CC1. The second instructor was an adjunct assistant professor at CC2. The third faculty, a mathematics educator from a senior CUNY college, did not teach any of the sections but observed random samples of sessions to ensure an alignment of the design and implementation.

The CC1 *fulltime faculty* taught both control and experimental groups in pre-algebra in Fall 2010 and algebra in Spring 2011. The CC2 *adjunct faculty* taught both pre-algebra sections in Fall 2010, but could only teach the experimental algebra in Spring 2011 section which had a load of 6 teaching hours. Because union regulations do not allow adjuncts to teach more than 9 hours per week, the CC2 instructor could not therefore teach the algebra control group in the same semester as the experimental group. That's why we had decided, a year earlier, to collect the survey and exams data of the control group in spring 2010. We chose to use the data generated during that semester for the algebra control group data in order to minimize the possible difference of instructors and semester effects (Table 3).

Table 3
Pre-Algebra and Algebra Experiments Schedule at CC1 and CC2

	CC1		**CC2**	
Total Students	**Control Group**	**Experimental Group**	**Control Group**	**Experimental Group**
n=192	**n=50**	**n=51**	**n=49**	**n=42**
Pre-Algebra	Fall 2010	Fall 2010	Fall 2010	Fall 2010
n=88	n=22	n=23	n=24	n=19
Algebra	Spring 2011	Spring 2011	Spring 2010*	Spring 2011
n=104	n=28	n=28	n=25	n=23

* See 3.11 sub-paragraphs above

3.12 Students. The sample represented by students in these gateway classes was also of convenience since the investigators taught these courses. However, the classes selected for intervention **(**technology-virtual manipulatives) and control (lecture-traditional) were randomly selected by the students who were not aware of this research study prior to registering. On the first day of classes, all students (n = 192) received a detailed explanation of the purpose of the study. The two faculty explained the content of the IRB consent form before they signed it. Students were given the option of not participating in the study or switching to a different section. None of the students elected to switch sections, or withdraw from the study at either campus. Moreover, when signing the consent forms, students in the experimental groups were given the option to be excluded from the audio or videotape recordings. The investigators honored the students' elections throughout the project (Table 3).

3.2 Treatment

The designed pre-algebra modules comprised integers, fractions, decimals, ratio and percent, while the algebra concepts studied were polynomial operations, factoring, functions and equation solving. These concepts represent more than 50% of the gateway mathematics curriculum. Gateway mathematics courses are taken by students who enter these community colleges without the necessary background to directly enroll into college level mathematics courses. While they bear no credits at CC2, they carry one credit in pre-algebra and two credits in algebra at CC1.

All pre-algebra and algebra concepts were taught to the experimental groups using the virtual manipulatives modules in a classroom using laptops (at CC1) or in a computer lab (at CC2). The same concepts were taught to the respective control groups using more traditional methods of teaching without reference to the virtual manipulatives. At CC1, the pre-algebra treatments for both control and experimental groups took place during the 2010 fall semester, while the algebra treatments were done in spring 2011 (Table 4). Similar procedures took place at CC2 with a slight change in the control group pre-treatment algebra data collected in spring 2010, a year earlier than the post treatment data. This slight change in the implementation was the result of complex programming issues involving teaching loads for adjuncts as explained earlier in the *3.11* subparagraph.

Table 4 ***Experimental and Control Groups Treatments at CC1 and CC2***

	CC1		**CC2**	
	Control Group Fall 2010	**Experimental Group Fall 2010**	**Control Group Fall 2010**	**Experimental Group Fall 2010**
Pre-Algebra topics studied • Fractions • Decimals • Ratio and Percent • Integers	**No use** of Laptops or Internet to introduce concepts	Use of virtual manipulatives modules from **NLVM** in a regular classroom with Laptops, Internet access and/or Software	**No use** of Laptops or Internet to introduce concepts	Use of virtual manipulatives modules from **NLVM** in a Computer Lab with Internet access and/or Software
	Data collected for both groups **Quantitative:** COMPASS results; Four Assessment Tests; and Final Exams. **Qualitative:** Entrance and Exit Surveys on mathematics attitudes and confidence in doing math; Informal Interviews with some students; Weekly Reflections; and Mathematical Autobiographies.			
	Control Group Spring 2011	**Experimental Group Spring 2011**	**Control Group Spring 2010**	**Experimental Group Spring 2011**
Algebra topics studied • Polynomial Operations • Factoring • Functions and Solving Linear Equations	**No use** of laptops or Internet to introduce concepts	Use of virtual manipulatives modules from **NLVM** in a regular classroom with Laptops, Internet access and/or Software	**No use** of Laptops or Internet to introduce concepts	Use of virtual manipulatives modules from **NLVM** in a Computer Lab with Internet access and/or Software
	Data collected for both groups **Quantitative:** Three assessment tests; and Final Exams. **Qualitative:** Entrance and Exit Surveys on mathematics attitudes and confidence in doing math (based on Fennema-Sherman scale); Informal Interviews with some students; Weekly Reflections; and Mathematical Autobiographies.			

3.21 Sample Pre-algebra and Algebra Activities. **Figure 1** below is a screen shot of the work of one pre-algebra student in the experimental group with prime factorization. Students were asked to perform first the prime factorization of 18 and 24, and then find the greatest common factor (GCF) and least common multiple (LCM) of the two numbers. **Figures 2 and 3** below are screen shots of one algebra student in the experimental group showing the first and final steps of solving a first degree equation with the variable on both sides. The student was asked to solve the equation: -2x + 5 = -x + 3. Students used virtual manipulatives to understand concepts and to practice in the following areas: operations with integers, multiplication of two binomials, graphing and solving linear equations.

Figures 1, 2, 3; Virtual manipulations in Pre-algebra and Algebra

1. Pre-Algebra-Screen Shot of a Student's Work with Prime Factorization ***2. Algebra-Screen shot of a Student's Work with Equation Solving*** ***3. Algebra: Final Screen Shot of a Student's Work with Equation Solving***

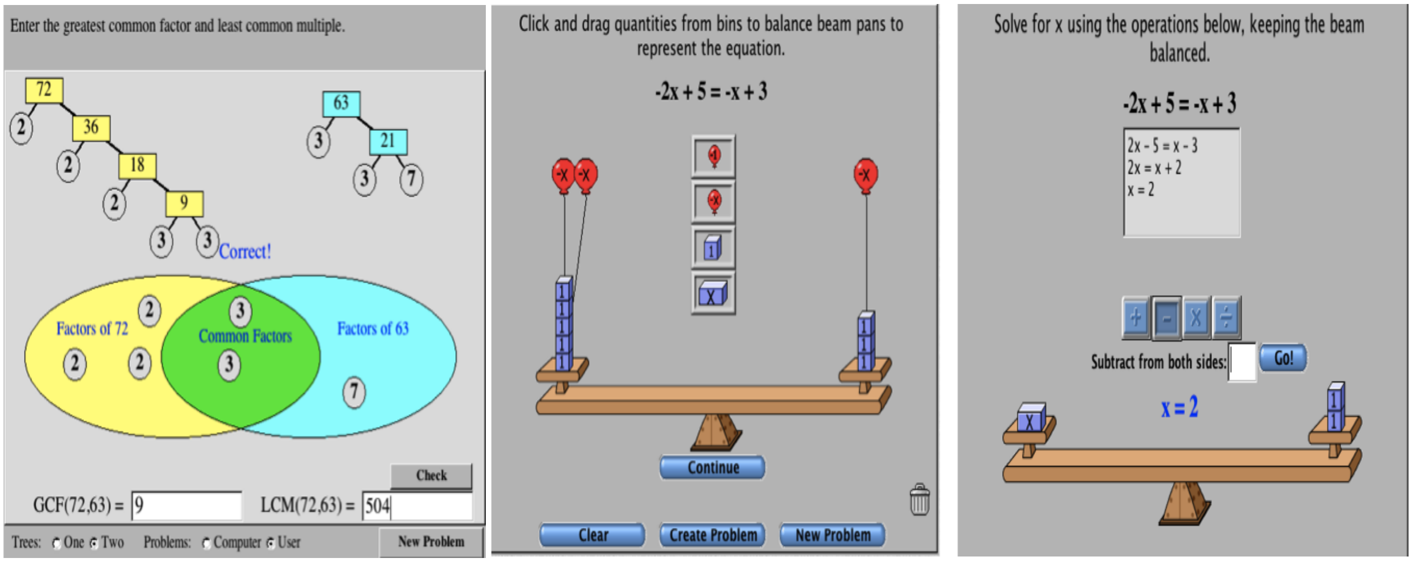
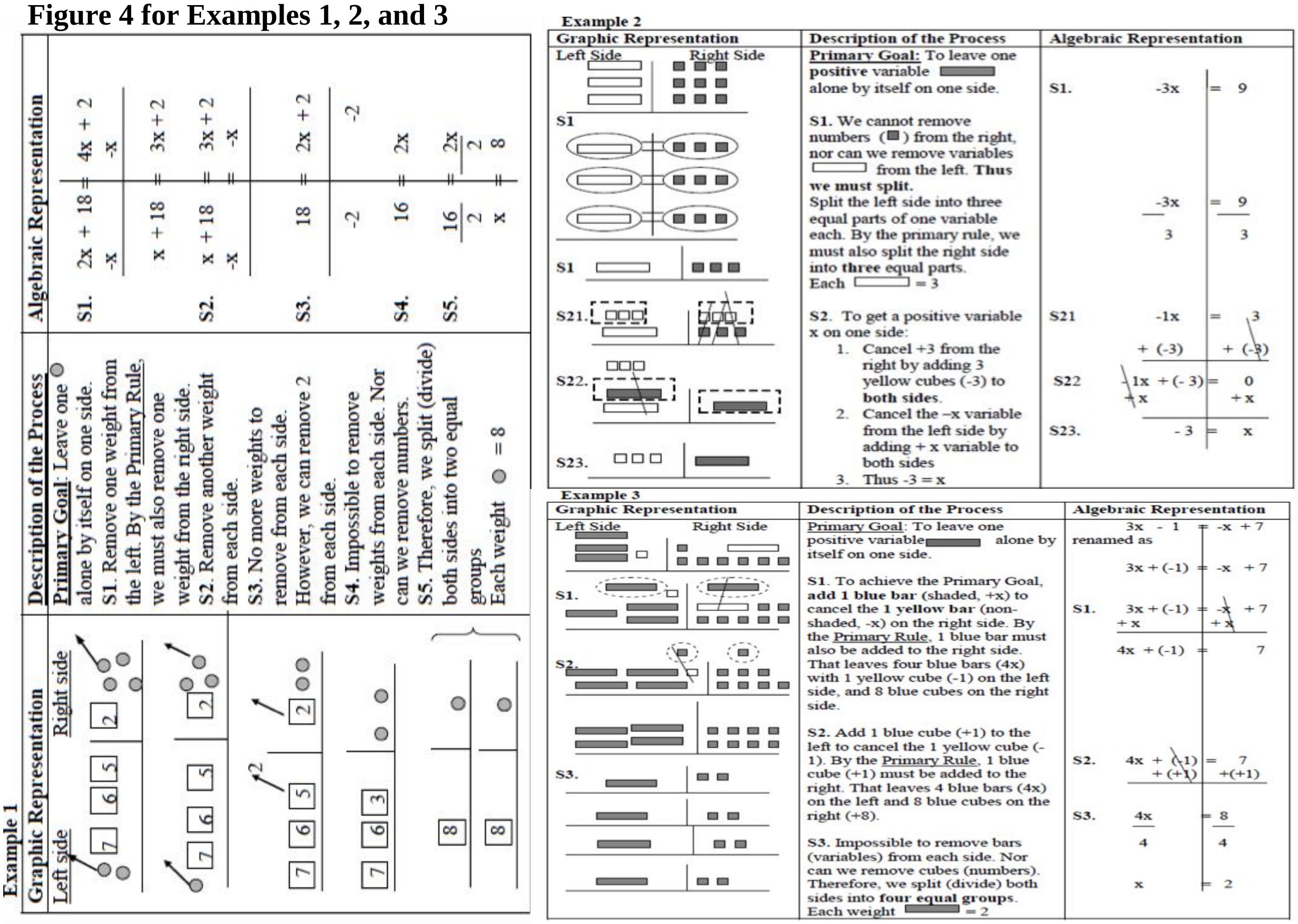


Figure 4 for Examples 1, 2, and 3



3.22 Transfer of Process into Pen and Paper. To solve the given equation, the student performed the following steps: (a) multiply both sides of the equation by -1 in order to change all the signs; (b) Add 5 to both sides of the equation; (c) subtract x from both sides of the equation. As we observe from the above screen shots, the student's screen displays both the manipulatives as well as the mathematical notation of all the steps the student undertook. The "primary goal" of the manipulation is to "*get rid of all variables except of one positive variable which has to be alone by itself on one side of the equation.*" The student decides then how to get there as long as he/she respects the "Primary Rule: *Whatever is done on one side needs to be done on the other side*." By drawing and writing the steps down, the students learn how to ultimately solve linear equation using symbolic notation. Students were therefore encouraged to use the following model as they worked with the virtual manipulatives. The drawings were especially important when they solved equations without the computers. Three examples of how manipulations are represented graphically (drawn) and carried out symbolically according to Bruner's stages of representation are shown in Figures 4 with Examples 1, 2, and 3.

In Figure 4-Example 1, students solve a multi-step equation with addition, multiplication, and the variable on both sides of the equation. In Figure 4-Example 2, students solve a "One-step" equation involving a multiplication and a negative integer. Finally, in Figure 4-Example 3, the steps of how students solved a "Multi-Step" equation with addition, multiplication, and both positive and negative variables on both sides of the equation are illustrated.

3.3 Data Collection

3.31 Pre-Algebra Pretests. Results to the initial placement COMPASS were used to determine the level of each pre-algebra class prior to the study. The COMPASS, an ACT-designed assessment (also known as the CUNY Assessment Test [CAT] in Mathematics), is an untimed, multiple-choice, computer-based test that measures students' knowledge of a number of topics in mathematics. The test covers progressively advanced topics with placement into more advanced mathematics or mathematics-related courses based on results of the last three sections of the test. The test draws questions from four sections: numerical skills/pre-algebra, algebra, college algebra, and trigonometry. Numerical skills/pre-algebra questions range from basic math concepts and skills (integers, fractions, and decimals) to the knowledge and skills that are required in an entry-level algebra course (absolute values, percentages, and exponents). The algebra items are questions from elementary and intermediate algebra (equations, polynomials, formula manipulations, and algebraic expressions). No two tests are the same; questions are assigned randomly from the four sections, adapting to your test-taking experience (http://cunymath.cuny.edu/COMPASS.php).

Reliability and validity measures were not available for the COMPASS test. The ACT argues however that "The COMPASS tests measure the skills and knowledge students need to succeed in specific courses. Students who have the skills and knowledge necessary to succeed in specific courses are likely to perform satisfactorily on the COMPASS tests, and students without those skills are not" ([1]).

Not all COMPASS scores were available. At CC1, the scores of 17 students for the experimental group and 22 students for the control group were available. At CC2, the scores of 18 students for the experimental group and 22 students for the control group were available. None of the students in the study, in pre-algebra, scored above 30, considered as the passing score for the COMPASS at both campuses.

Table 5 shows that, at CC1, the difference in performance on the COMPASS exam between the control and experimental groups was not significant, while at CC2, the control group scored significantly higher than the experimental group. These results on the Compass test were by sheer chance since the sample represented by students in these gateway classes was of convenience. We decided to continue the experiment at CC2 despite the fact that the control group had higher scores on the COMPASS exam for three reasons: (1) it was not possible for the instructor to change his sections since the semester had already begun when we got the COMPASS results; (2) we hypothesized that the virtual manipulatives could play the role of an equalizer that could allow experimental group students to reduce the gap in performance; and (3) on an informal pre-assessment given at the beginning of the course to all four groups at both campuses, the results were dismal for all four groups. The two instructors have had a great deal of experience teaching these courses and made the point that they always start these classes with the thought that students have very limited prior knowledge anyway.

Table 5
Pretest Compass Mean Scores - Experimental vs. Control

	CC1		**CC2**	
	Experimental Group (n=17)	**Control Group (n =22)**	**Experimental Group (n=18)**	**Control Group (n =22)**
Mean	21.53	19.45	17.33	23.45
Std. Deviation	4.543	3.334	23.45	3.203
t	1.646*		-.6.425**	

*p>.05 not significant **p<.05 significant

3.32 Quantitative Data. Multiple sources of quantitative and qualitative data were used to test the hypotheses. The quantitative data collected in the pre-algebra classes consisted of: Students' scores in the four achievement tests at the conclusion of each of the studied units: fractions (Exam 1); decimals (Exam 2); percents (Exam 3); integers (Exam 4); and in the uniform pre-algebra final examination given to all students in each college's pre-algebra classes (different examinations were used by the respective mathematics departments at CC1 and CC2). **At CC1,** the final examination consisting of 25 questions covering all the studied topics and designed by the mathematics department was administered to all sections, including those not taught by the researchers, during the last week of the pre-algebra classes and was a summative evaluation of individual students' learning. **At CC2**, a similar final examination, consisting of 20 questions was administered under the same conditions. The mid achievement tests however, were designed by the two instructors/researchers based on the topics they taught. The two agreed on the format and the type of questions to use for each exam.

The quantitative data collected in the algebra classes consisted of students' scores in the achievement tests at the conclusion of the units on polynomial operations (Exam 1); factoring (Exam 2); functions and solving linear equations (Exam 3); and in the uniform algebra final examination given to all students in each college's algebra classes (different examinations were used by the respective mathematics department at CC1 and CC2). Reliability and validity measures were available for neither the pre-algebra nor the algebra posttests. Similar uniform final exams are designed and given each year by the Mathematics Departments at each college.

3.33 Qualitative Data. A variety qualitative data were collected. All pre-algebra and algebra students provided responses to a 24-question Likert scale survey on attitudes toward and confidence in doing mathematics. The questionnaires were based on two subscales of a revised version of the Fennema-Sherman Mathematics Attitude Scales by Hackett and Betz [17]. Identical questionnaires were given to all participating students at the beginning and end of each class. As a certain percentage of the students (approximately 20%) dropped out of the classes before conclusion, not all students who filled the questionnaire at the beginning of the course filled it at the end. The survey is available in the appendix below.

The first category of the survey, the confidence category (Questions 1-12), attempts to measure students' confidence in approaching mathematical tasks and belief in their ability to successfully complete these tasks. In the first six questions (1-6), a response of 'Strongly Agree' would indicate the highest level of confidence, while a student's response of 'Strongly Disagree' would indicate a lowest level of confidence. In questions 7-12, a response of 'Strongly Agree' would indicate the lowest level of confidence, while a response of 'Strongly Disagree' would indicate a high level of confidence.
The second category, the anxiety category (Questions 13-24), attempts to measure students' level of anxiety and the effect of this anxiety on their performance in mathematics. In the first six questions 13-18 (for instance, Math does not scare me at all), a student who responded by selecting 'Strongly Agree' would have a low level of anxiety about mathematics, whereas a student who responded by selecting 'Strongly Disagree' would have a high level of anxiety. Similarly to the confidence category, in questions 19-24, a response of 'Strongly Agree' would indicate the highest level of anxiety, while a student's response of 'Strongly Disagree' would indicate a low level of anxiety. In entering the data into SPSS, all responses in questions 7-12 and 19-24 were reversed to make all responses "Strongly Disagree –Strongly Agree" be represented by the same values 1-5.

Alpha reliability analyses were performed on the summed scores for the scales of confidence (C) and mathematics anxiety (A) with 12 items each. Using the "Reliability" procedure in the statistics software SPSS, we found internal consistency estimates of the reliability of scores to be .9297 on the confidence category, .8802 on the mathematics anxiety, and .9422 on the whole scale. These results are very similar to the ones obtained by Fennema and Sherman [12] and Mulhern and Rae [24]. The results suggest that the single composite score and each of the subscale scores have high reliability.

Informal interviews with students coupled with weekly reflections written by students provided researchers with insights into their mathematical difficulties and attitudes toward use of technology in learning mathematics. Some students also produced mathematical autobiographies that provided the investigators with insight into the students' home culture and exposure to learning of mathematics in their elementary, middle and high school years.

3.4 Data Analysis

A mixed-methods approach enabled the researchers to triangulate data. Triangulation of data occurs when several different methods of data-gathering are used to bear on a particular research question or topic. It also enabled us to determine if there were significant patterns and relationships in our quantitative evidence that needed to be pursued in greater depth using more qualitative evidence or through different quantitative measures. One advantage of a triangulated mixed-method approach to data-gathering and analysis is that it increases the validity of the results. Another is that the qualitative data can elucidate and enrich the quantitative data. In sum, a mixed-methods approach provides, "rich opportunities for cross-validating and cross-fertilizing… procedures, findings, and theories."

A one-way analysis of variance (ANOVA) design was used to investigate students' performance on pre-algebra and algebra concepts with (experimental group) and without (control group) the use of virtual manipulatives. Results of the Fennema-Sherman Mathematics Attitude Scales were analyzed as a part of the qualitative analysis of students' confidence in and attitude toward mathematics. The recorded events and students' journals were analyzed by the researchers who sought patterns of learning and compared the learning with virtual manipulatives to the learning in lecture-format classrooms.

4. Results

4.1 CC1 Pre-Algebra

Table 6 shows that for the CC1 Pre-Algebra classes, the experimental group outperformed the control group in two out of five assessments at the .05 level: Exam 2 (Decimals) and Exam 3 (Ratio and Percents), but not in Exam 1 (Fractions), Exam 4 (Integers), and in the Final Examination.

Table 6

CC1 Pre-Algebra (Fall 2010) Post-Treatment Means on All Assessments-Experimental vs. Control

		Mean	**Std. Dev.**	**n**	**p**
Exam 1	Experimental	67.7391	18.0384	23	$p > .05$**
	Control	58.3182	17.8069	22	
Exam 2	Experimental	76.1905	17.8371	21	$p < .05$*
	Control	63.4000	14.9961	20	
Exam 3	Experimental	54.1905	22.7104	21	$p < .05$*
	Control	34.7500	23.0434	16	
Exam 4	Experimental	59.7619	22.2866	21	$p > .05$**
	Control	46.0625	20.4531	16	
Final Exam	Experimental	60.2125	22.1257	20	$p > .05$**
	Control	45.3929	23.6521	14	

*Significant in favor of the Experimental group; ** Not Significant

4.2 CC2 Pre-Algebra

Table 7 reveals that for the CC2 Pre-Algebra classes, the mean scores for the control group are higher than the mean scores of the experimental group in all five assessments: Exam1, Exam 2, Exam 3, Exam 4 and the Final Examination. Notice that Exam 4 has the smallest mean difference in favor of the control group. The graph shows significant mean differences in all the five assessments in favor of the control group, ($p < .05$). Factors that seem to have contributed to the control group's better performance over the experimental group include time effect (8:00 am class vs. 10:00 am); and age effect (younger students vs. mature students).

Table 7
CC2 Pre-Algebra Post Treatment Means on All Assessments Experimental vs. Control

		Mean	Std. Dev.	n	
Exam 1	Experimental	56.4211	20.3369	19	P<.05*
	Control	76.7500	11.2607	24	
Exam 2	Experimental	62.8125	19.7018	16	P<.05*
	Control	78.8261	12.2053	23	
Exam 3	Experimental	68.1765	14.1476	17	P<.05*
	Control	80.2632	9.6370	19	
Exam 4	Experimental	73.5714	17.0687	14	P<.05*
	Control	82.2727	9.6913	22	
Final Exam	Experimental	65.8125	23.1177	16	P<.05*
	Control	78.7000	12.1400	20	

*Significant in favor of the Control group

4.3 CC1 Algebra

Table 8 shows that for the CC1 algebra classes, the mean of the experimental group is higher than the mean of the control group in three of the four assessments: Exam 1, Exam 3 and the Final Exam. However, the mean differences in these three exams between the experimental and control were not all significant. The graph shows that the mean difference in Exam 3 is the only assessment that resulted in a significant finding ($p < .05$).

Table 8
CC1 Algebra Post Treatment Mean Comparisons on All Assessments-Experimental vs. Control

		Mean	Std. Dev.	n	
Exam 1	Experimental	65.9286	23.9164	28	p > .05**
	Control	56.9643	30.0166	28	
Exam 2	Experimental	69.3571	28.5362	28	p > .05**
	Control	79.8333	23.0551	24	
Exam 3	Experimental	72.8800	20.2471	25	p < .05*
	Control	55.6190	26.5207	21	
Final	Experimental	52.1600	21.7269	25	p > .05**
	Control	48.1053	24.1452	19	

*Significant in favor of the Experimental group; ** Not Significant

4.4 CC2 Algebra

Table 9 shows that for the CC2 algebra classes, the experimental group had a higher mean performance than the control group in all four assessments. However, only the mean difference in Exam 2 was significant in favor of the experimental group ($p < .05$).

4.5 Students' Attitudes and Confidence

From the pre-survey to the post-survey, the overall response mean in attitude and confidence significantly changed for only one of the eight groups: the CC2 Algebra Control Group ($p < .05$). When all questions were studied individually however, an increase in the mean responses was noted for all groups for each one of the questions. Interestingly, for two of the “confidence” questions, “I am no good at math,” and “Math has always been my worst subject,” a significant

Table 9
CC2 Algebra Post Treatment Mean Comparisons on All Assessments-Experimental vs. Control

		Mean	**Std. Dev.**	**n**	**Significance**
Exam1	Experimental	65.0000	27.5780	23	p > .05**
	Control	57.6800	21.4218	25	
Exam2	Experimental	71.3684	25.2019	19	p < .05*
	Control	56.0909	20.1823	22	
Exam3	Experimental	69.7895	25.8963	19	p > .05**
	Control	60.5500	19.4733	20	
Final	Experimental	67.3500	24.6177	20	p > .05**
	Control	60.0000	18.9816	21	

*Significant in favor of the experimental group; ** Not significant

change ($p < .05$) was found in both experimental pre-algebra classes at CC1 and CC2. This result led us to believe that the confidence in doing mathematics level of students in the Pre-Algebra Experimental groups' grew more significantly after using the virtual manipulatives than the confidence level of students in the pre-algebra control groups' at CC1 and CC2 after being taught traditionally. For the algebra groups in general, no significant changes were noted from the pre- to post-surveys on any of the attitudes and confidence indicators. The control groups at both CC1 and CC2 displayed however, a more positive attitude and had a higher confidence level than any of the other groups at the beginning of the algebra courses. Since algebra students have already passed pre-algebra, they perhaps believe more in their ability to do mathematics than pre-algebra students.

5. Discussion

Working with virtual manipulatives did help somewhat the experimental group understand mathematical concepts and clear some long-held mathematical misconceptions. Even if the data revealed test scores were statistically different in favor of the control group at CC2, the qualitative data indicated no negative impacts of the use of the Internet and software-based activities into the classroom, especially given the results obtained at CC1 and students reflections and actions outside the classroom. The learning with technology allowed the more conscientious students to repeat the lessons at home, on their computer, until they mastered the concepts. The computer software provided the students with many new practice exercises and instant feedback [20]. All students were motivated to go online and work with the virtual manipulatives. The technology-infused classes led students to better group work and co-teaching. There was a change in the dynamic of the experimental classes as students in these classes volunteered to be assistant teachers and to help other students. The teacher was not the only expert in the classroom making these classes more student-centered [30]. Students learned and practiced many concepts on their own, not needing the teacher to come verify their answers. In the tedious work of subtraction of integers for instance, the concept of zero-pair was essential to understanding the meaning of subtraction [14], [18]. The technology enabled students to try different combinations and strategies on what type of zero-pair to add, and check the results immediately to validate their answers, something they could not do without technology [14], [34]. The computer software provided the students with many new practice exercises and instant feedback.
In instances when the technology was not available as in the control groups, students struggled to understand the concepts taught while waiting for better teacher explanation.

On the downside, the very demanding syllabus and class tempo, combined with many students' lack of math preparedness and high absenteeism or tardiness, prevented some students from gaining the deeper conceptual understanding and effect a smooth transition from the iconic to the abstract representation. It appears that few students "played" with manipulatives outside the lab, despite repeated recommendations to do so. We had indeed expected maturity to play a role in the experiment, but not in the way it played out. We expected to see no "attitudinal interference" [27] and we didn't. We hypothesized that college students would not adopt middle and high school students' attitude that manipulatives are for play and for young children and consequently block their full engagement when working with objects [27]. They did not. What we did not take into account however, was what we would call "maturity interference" caused mainly by factors students did not control. At CC2 for instance, the experimental group, which started at 8:00 am, attracted much younger students with fewer domestic responsibilities. They had a more lackadaisical attitude towards learning in general. Many of them were single men working night jobs and arrived late to class or lab. On the other hand, the control group, which started at 10:00 am, was mostly composed of single mothers and parents who preferred the later class to cater to child-rearing arrangements. Students in this group displayed a more mature attitude toward learning mathematics than the experimental students and demonstrated a higher motivation and sense of responsibility. According to the Council for Adult and Exponential Learning (CAEL) [9] such outcomes are not surprising as mature individuals are more motivated and have more life and work experience from which they have learned.

The infusion of descriptive writing as illustrated in Figure 4 with examples 1, 2, and 3 for the concepts of solving equation provided multiple benefits. Students not only drew representations of the process but also reflected in writing the symbolic or algebraic representations. Reflective writing enables students of all ages to think of what they are doing rather than repeating automatically something they memorized, leading them to internalize the process and understand concepts ([37], [10], [15]). The addition of this step led us to believe that Bruner's three stages of representation could be modified to include a fourth stage, that of descriptive writing, which seems to happen before the symbolic stage. Using writing to assist in the assimilation of procedural knowledge and concept development was inspired by the work of Vygotsky [40] who believed that writing due to its planned and conscious nature will increase students' structural understanding of mathematics.

6 Conclusions and Recommendations

An analysis of the qualitative data helped us explain the mixed results that were obtained with the analysis of the quantitative data. Virtual manipulatives were useful for students learning basic mathematics concepts. Students expressed this usefulness clearly in their reflections, in face-to-face interviews and through their answers and in the surveys on attitudes toward and confidence in doing mathematics.

The experimental group students overcame their initial mathematics misconceptions with less difficulty than the students in the control group. They found the classes with virtual manipulatives very exciting. With the virtual manipulatives, they had the opportunity to try numerous times and get instant feedback before the instructor could turn his/her attention to them. This process of trial and error often enabled them to make a conjecture on a possible solution which then had to be validated by the instructor.

A primary finding seems to be that the virtual manipulatives appear to be more useful in teaching pre-algebra remedial courses than an algebra remedial courses, for the simple reasons that the modules for pre-algebra cover most topics (fractions, decimals, percentages, ratios and proportions, operations with integers and solving linear equations) whereas the virtual modules for algebra do not cover inequalities, factoring of polynomials and solving quadratic equations. The algebra curricula at both CC1 and CC2 are also accelerated. Teaching a topic with virtual manipulatives takes much longer than teaching the same topic in a traditional lecture-style class. Consequently, learning of algebra with virtual manipulatives requires more hours than a typical one-semester class. Moreover, by its nature, algebra is more abstract than pre-algebra. Students learning algebra concepts are not helped as much as students learning pre-algebra with virtual manipulatives. Future studies are needed to determine the longer-term utility of learning remedial mathematics with virtual manipulatives. We do assume that students who learned and mastered mathematics concepts with virtual manipulatives will retain the concepts longer.

Another finding that needs further research is the inclusion of a fourth stage on Bruner's three stages of representation. Did the use of writing in parallel with that of manipulative devices to increase student's mathematical development increase students' procedural proficiency, in what Haapasalo and Kadijevich ([16]) called "simultaneous action?" Throughout the treatment, both control and experimental groups were provided with opportunities to practice using skills. The use of conceptual procedures was supplemented with a plethora of exercises for skill development. Future studies will have the challenge to establish evidence on the role of writing in promoting, "conceptual development or increased mathematical maturity" ([31]).

Should virtual manipulatives be an integral part of teaching remedial mathematics in community colleges? To answer this question we have to consider the investment in technology, the training of faculty and the willingness of the faculty to embrace technology in teaching. In order to maximize the effectiveness of using virtual manipulatives, we recommend the following:

1. Provide an intensive two-week workshop series using virtual manipulatives to those students who failed the pre-algebra COMPASS placement test and those students who are multiple repeaters of the remedial pre-algebra course;
2. Provide training in the use of technology to faculty members who will be conducting those workshops.

7 Significance of the study

With close to 80 percent of incoming freshmen at the two community colleges in this study needing to take at least one remedial course, the issue that arises is about the solution to reducing such high number. As the number of students enrolling in community colleges increase, so does the urgency to address their needs. Students in urban areas, especially minority students, face many challenges to be college ready. How students can be best prepared them for entry into College? How best can we help students who need remediation? Policymakers see algebra as the central problem. The sharp falloff in mathematics achievement in the U.S. begins as students reach late middle school, where, for more and more students, algebra coursework begins ([25]). For the National Assessment of Educational Progress (NAEP), the earlier students in middle school take their first year algebra for instance, the higher their average mathematics proficiency by the time they reach grade 12 ([11]). Among African-American and Hispanic students with mathematics preparation at least through Algebra II, the differences in college graduation rates versus the student population in general are half as large as the differences for students who do not complete Algebra II ([25], p.xiii).

CC1 and CC2 being both Hispanic Serving Institutions, are in a favorable position to respond to calls to steer changes in the teaching and learning of mathematics, especially in the Bronx and the New York City metropolitan area. Located in the Bronx, where close to 90% of all public school students are Black or Hispanic, the two colleges have expressed a special commitment to improve the education of all students in the borough. Virtual manipulatives could play a significant role in the teaching of remedial math classes (pre-algebra and algebra) to community college students and to middle and high school students. The use of technology-rich and easy-to-use materials can be appealing to college instructors of these classes, more than hands-on manipulatives. Moreover, success in pre-algebra and algebra courses seems to be a strong indicator of future college success. Success in these courses bolsters students' attitude and confidence toward taking future mathematics classes, increases their chance of graduating from community college and decreases the dropout rate.

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9 APPENDIX

Questionnaire on Attitude towards Mathematics with possible Responses as SD, D, N, A, SA

Scale	Item
1	Generally I have felt secure about attempting mathematics
2	I am sure I could do advanced work in mathematics
3	I am sure that I can learn mathematics
4	I think I could handle more difficult mathematics
5	I can get good grades in mathematics
6	I have a lot of self-confidence when it comes to math
7	I'm no good at math
8	I don't think I could do advanced mathematics
9	I'm not the type to do well in math
10	For some reason even though I study, math seems unusually hard for me
11	Most subjects I can handle OK. but I have a knack of mucking up math
12	Math has been my worse subject
13	Math doesn't scare me at all
14	It wouldn't bother me at all to take more math courses
15	I haven't usually worried about being able to solve math problems
16	I almost never have got nervous during a math test
17	I usually have been at ease during math tests
18	I usually have been at ease in math classes
19	Mathematics usually makes me feel uncomfortable and nervous
20	Mathematics makes me feel uncomfortable, restless, irritable, and impatient
21	I get a sinking feeling when I think of trying math problems
22	My mind goes blank and I am unable to think clearly when working mathematics
23	A math test would scare me
24	Math makes me feel uneasy and confused